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"Identifying Sorting - In Theory"

by

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Identifying Sorting - In Theory*

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Abstract

We argue that using wage data alone, it is virtually impossible to identify whether Assortative Matching between worker and firm types is positive or negative. In standard competitive matching models the wages are determined by the *marginal* contribution of a worker, and the marginal contribution might be higher or lower for low productivity firms depending on the production function. For every production function that induces positive sorting we can find a production function that induces negative sorting but generates identical wages. This arises even when we allow for non-competitive mismatch, for example due to search frictions. Even though we cannot identify the *sign* of the sorting, we can identify the *strength*, i.e., the magnitude of the cross-partial, and the associated welfare loss. While we show analytically that standard fixed effects regressions are not suitable to recover the strength of sorting, we propose an alternative procedure that measures the strength of sorting in the presence of search frictions independent of the sign of the sorting.

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1 Introduction

Sorting of workers to jobs matters for the efficient production of output in the economy. If there are strong complementarities or substitutes between workers and jobs, the exact allocation has large efficiency implications. In contrast, when complementarities are nearly absent, not much output is lost from the random allocation of workers to jobs. This is important for policy, for example whether we want to design an unemployment insurance program that provides incentives for workers to look for the "right" job instead of accepting the first offer (see for example Acemoglu and Shimer 1999). There are also profound implications for redistribution policies. In the presence of strong sorting, redistribution through mismatching leads to substantial distortions whereas the distortions are minimal when sorting is weak. And subsidies to education will in the presence of strong sorting lead to increased competition between workers, thus transferring some of the subsidy rents from workers to firms.

Given the importance of sorting, a large body of recent empirical literature has estimated whether sorting is positive or negative. In part, this renewed interest has been catalyzed by the availability of worker-firm match data. Several empirical papers (Abowd, Kramarz, and Margolis (1999), Abowd, Kramarz, Lengermann, and Perez-Duarte (2004)) find an insignificant or even negative correlation in fixed effects between worker and firm types. This result has been replicated for a number of countries such as France, US, Denmark and Brazil. The result is taken as indication that Positive Assortative Matching between workers and firms does not play a role in the labor market.

On the other hand, recent work reveals that a structural labor search model that has strong complementarities and thus induces Positive Assortative Matching in equilibrium can nevertheless generate smaller and even negative correlations in the fixed effects of workers and firms (Gautier and Teulings (2006), Lise, Meghir and Robin (2008), Lopes de Melo (2008) and Lentz (2008)). In some cases this is attributed to the non-linear structure of the wage setting that is not picked up in the regression specification. Other work by Gabaix and Landier (2008) argues that sorting of CEOs has become more important as is reflected in the increased wages.

In this paper we address the issue of identification of sorting in a simple theoretical framework and obtain the following results. First, in the frictionless matching model (Becker 1973), we show that identifying whether sorting is positive or negative is impossible using wage data alone. To see this, note that in this model more productive workers always earn higher wages and more productive firms always make more profits. Under Positive Assortative Matching (PAM), the more productive firm also has the highest marginal product from labor, i.e. the cross-partial is positive. This implies that high type firms hire high type workers and they pay high wages. Under Negative Assortative Matching (NAM) instead, low productive firms have a comparative advantage from hiring more productive workers, i.e.

the cross-partial is negative.[1] As a result, high type firms pay lower wages. By ranking firms according to the wages they pay, we do not identify the most productive firm. Without any additional data on the profitability of each job, it is impossible to identify whether sorting is positive or negative. This is true even if we consider wages off the equilibrium path.

Second, we explicitly allow for mismatch to occur in equilibrium due to search frictions. We find that the first-order effect is that wages of a given worker have an inverse U-shape around the optimal allocation, the "bliss point". Under mismatch with a relatively bad firm, wages increase as the firm type increases because the firm moves closer to the bliss-point, whereas wages decrease once the worker is mismatched with too good a firm because better firms move further from the bliss-point. Higher productivity firms have to be compensated for their willingness to match with a "bad" worker because it destroys their opportunity to match with a "good" worker. This leads to the bliss-point in the pattern of compensation if a worker meets the "right" firm, rather then a wage schedule that is increasing everywhere in the type of firm. The net effect on a given worker's wages from increasing the firm type is therefore ambiguous and second-order. For the simplest, type-independent cost of delay, we explicitly show that the net effect is exactly equal to zero under some common specification. This version, also studied in Atakan (2006), is a close reformulation of Becker (1973). When search costs are type-dependent (as in Shimer and Smith (2000) for example), the net effect may still be either positive or negative. The additional component induced through type-dependence is proportional to the magnitude of the friction and, compared to the inverted U-shape, the net effect of firm type on wages is likely to be second-order and difficult to isolate in the data.

Third, and in spite of the fact that we cannot identify the *sign*, we develop a method that in theory allows for identification of the *strength* of sorting. Ultimately, efficiency properties depend on how big the complementarities/substitutes are. Identification derives from the distinct features of the range of wages a worker receives who has been observed repeatedly. First, the highest wage corresponds to her bliss-point. We use this to order the workers and obtain the type distribution. Likewise, we can obtain an order of the firms by the level of wages that they pay (this identifies those firms with the highest willingness to hire better workers - which is positively related to type under positive sorting and negatively under negative sorting). Second, the difference between the highest and the lowest wage is equivalent to the cost of search. Third, we calculate the loss due to mismatch over the range of wages, which from the theory can be expressed in terms of the absolute value of the cross-partial of the production function. Given that we know the cost of search, we can now obtain the strength of the cross-partial, or equivalently, the strength of sorting.

[1] We use the term comparative advantage to denote a larger absolute gain in output from matching with a better worker, rather than the stronger concept of a larger percentage gain as used e.g. in Sattinger (1975).

Before proceeding to the model, we briefly lay out the empirical issue. Abowd, Kramarz, and Margolis (1999) use a simple empirical measure of sorting that can be obtained by estimating a log-wage equation in which wages are a function of a worker fixed effect, a firm fixed effect, and an orthogonal error term

$$\log w_{it} = a_{it}\beta + \delta_i + \psi_{j(i,t)} + \varepsilon_{it} \tag{1}$$

where w_{it} denotes the wage, a_{it} are time varying observables of workers, δ_i is a worker fixed effect, ψ_j is the fixed effect of the firm j at which worker i is employed at time t, and ε_{it} is an orthogonal residual. That is, ψ_j captures the average effect that a firm has on the wages of the workers that are willing to match with it. The correlation between α_i and ψ_j in a given match is taken as an estimate of the degree of sorting. Abowd et al (2004) propose a bargaining procedure in which higher firms pay higher wages in line with (1) as a test of Becker's (1973) idea of matching.

We argue here that such a pay schedule is inconsistent with Becker's theory because every worker would like to match with the best firm. Becker's theory is inseparably linked to a theory of wages which prevents such overcrowding of workers at top firms. We point out the distinctive features of the underlying theory of wages, and extend them to a simple and tractable model with search frictions. We show in our theoretical exercise that the assumption that the firm effect is independent of the worker's type is theoretically not justified in this setting. In particular, for those workers who are matched with a firm that has a lower rank than their own, the wage increases when the firm type increases because the worker-firm "fit" improves. In contrast, workers who are matched with a lower ranked firm see a decrease in the wage when the firm becomes better because the worker-firm "fit" deteriorates.

A similar logic holds in the context of matching with frictions. With wage bargaining in a model with search frictions, the set of eligible partners is bounded by those matches where the match surplus is zero relative to continue searching. For all acceptable partners the surplus is positive. For a given worker type, the surplus goes to zero for a match both with too bad a firm and too good a firm. Any bargaining procedure that pays wages that are monotonic in the surplus will therefore result in wages being non-monotonic in firm type.

The goal of our analysis is to lay out this logic in the simplest possible environment. First, we consider the frictionless benchmark, and then we extend it in a straight-forward way to a model with frictions that allows worker and firm mismatch. Frictions are modeled in a two-stage set up: a stage of random matching is followed by a frictionless matching stage. The benefit of our modeling approach is that the main effects that drive the wage determination become clearly visible and highlights the forces, limitations, and possibilities that arise in estimations that are based solely on wage data. As such, it informs our understanding of the results obtained in more complicated infinite horizon steady-state models that preclude closed-form theoretical analysis but are often used for structural estimation.

2 The Frictionless Model

In order to make our point we start with the following very simple matching model following Becker (1973). There is a unit mass of worker and a unit mass of firms. Workers and firms are heterogeneous in terms of their productivity. Workers draw their type x from distribution $\Gamma(x)$ with smooth density $\gamma(x)$ on $[0,1]$. Firms draw their type y from distribution $\Upsilon(y)$ with smooth density $\upsilon(y)$ on $[0,1]$.

When types x and y form a match, they produce positive output $f(x,y) > 0$ whilst having an outside option of remaining unmatched. We assume that workers and firms can be ranked in terms of their productivity, i.e. $f_x > 0$ and $f_y > 0$. Then it is without loss of generality to index a worker by his *rank* in terms of productivity, i.e. by the fraction of workers that are less productive then him. Similarly, we can identify each firm by its rank in the distribution of firm productivities. This means that $\Gamma(.) = \Upsilon(.) = x$, i.e. the distributions are uniform.[2] Assume that workers who do not get matched obtain a payoff of zero, and since output is non-negative, all agents will prefer to match. If output of all matches is strictly positive, there will be a continuum of wage schedules that can support the same allocation, and we will assume that an exogenous bargaining procedure determines which split of surplus is implemented. We will denote this payoff by the constant $w_0 \geq 0$ that pins down the wage of the lowest worker type.

For the assignment of workers to firms the cross-partial of the production function is important. We do not restrict the sign of the cross-partial since this will be instrumental in determining whether there is positive or negative assortative matching. Denote by $\mathcal{F}$ be the class of all functions f that are monotonic: $f_x, f_y > 0$; and that have a monotonic marginal product: $f_{xy}(x,y)$ is either always positive or always negative.[3] The assumption that the cross-partial does not change sign allows us to unambiguously talk about positive or negative sorting. Production functions for which higher worker types always have a comparative advantage at better firms ($f_{xy} > 0$) are in set $\mathcal{F}^+ \subset \mathcal{F}$. Production functions for which higher worker types always have a comparative advantage at lower firms ($f_{xy} < 0$) are in set $\mathcal{F}^- \subset \mathcal{F}$.

To illustrate the implications of our analysis we will derive our results for the following examples of production functions

$$f^+(x,y) = \alpha x^\theta y^\theta + h(x) + g(y), \tag{2}$$

$$f^-(x,y) = \alpha x^\theta (1-y)^\theta + h(x) + g(y), \tag{3}$$

[2] The uniformity assumption is without loss of generality since the production function can be identified only up to a normalization: For a general $\Gamma(\cdot)$ and $\Upsilon(\cdot)$ and $f(\cdot,\cdot)$ we can alternatively consider a uniform type space in which only the ranking matters and an alternative production function $\tilde{f}(x,y) = f(\Gamma(x), \Upsilon(y))$.

[3] Later, in section 5 we discuss the virtues of relaxing this assumption.

where $g(.)$ and $h(.)$ are increasing functions and $\alpha \geq 0$ and $\theta > 0$ are parameters that indicate the strength of the complementarities. We assume that $g(y)$ is such that higher firms produce higher output even under the second specification. It is obvious that $f^+ \in \mathcal{F}^+$ and $f^- \in \mathcal{F}^-$.

An assignment of workers x to firms y is denoted by μ, i.e., the partner y of worker x is $\mu(x)$. In this part of the paper we assume a competitive matching market. A market equilibrium specifies an assignment between x's and y's and some wage schedule $w(x,y) \geq w_0$ that determines the split of output between the worker and the firm. The payoff to the worker is $w(x,y)$ and the payoff to the firm is $\pi(x,y) = f(x,y) - w(x,y) \geq 0$. Both workers and firms take the wage schedule as given. The tuple of functions (μ, w) is an equilibrium if no worker wants to switch to a different firm at the market wages, i.e.

$$w(x,\mu(x)) \geq w(x,y) \text{ for all } x \text{ and } y;$$

and no firm wishes to employ a different worker, i.e.

$$\pi(\mu^{-1}(y), y) \geq \pi(x,y) \text{ for all } x \text{ and } y.$$[4]

We derive the main prediction of Becker's (1973) model concerning the wages in the economy. In equilibrium, each firm y maximizes profits, taking the wage schedule as given:

$$\max_x f(x,y) - w(x,y).$$

This yields the first order condition

$$f_x(x,y) - \frac{dw(x,y)}{dx} = 0. \tag{4}$$

Let $w^\star(x)$ be the equilibrium wage of worker x. Integrating (4) along the equilibrium path yields

$$w^\star(x) = \int_0^x f_x(\tilde{x}, \mu(\tilde{x}))d\tilde{x} + w_0, \tag{5}$$

where the constant of integration is pinned down by w_0. Observe that the worker obtains exactly his marginal product along the equilibrium allocation. Therefore, equilibrium profits of type y are given by output minus the wage $w^\star$ with the optimal worker $\mu^{-1}(y)$. This can be re-written as

$$\pi^\star(y) = \int_0^y f_y(\mu^{-1}(\tilde{y}), \tilde{y})d\tilde{y} + f(0,0) - w_0 \tag{6}$$

Furthermore, we know from Becker's analysis that matching is positive assortative when the production function is supermodular ($f_{xy} > 0$), in which case $\mu(x) = x$. Under submodularity ($f_{xy} < 0$) in

[4] It is well-known that a strict cross-partial yields a one-to-one mapping $\mu(\cdot)$ in equilibrium. In general $\mu(\cdot)$ is a correspondence, with the equilibrium definition extended to all pairs in that correspondence.

equilibrium the matching is negative assortative and $\mu(x) = 1 - x$. With this in mind, we show that in this simple competitive model the direction of sorting - i.e. the sign of the cross-partial - cannot be identified from wage data. We first show this result on the equilibrium path, then we show it off the equilibrium path. In the next section we build an extended model with search frictions where the wages off the equilibrium path actually arise.

2.1 On the equilibrium path

We will first illustrate the result by considering our restricted class of production functions outlined above and then present the general theorem. Suppose the underlying production technology is not known and the true technology is either one of the two example technologies f^+ given in (2) or f^- given in (3). By (5) the wages under f^+ are

$$\begin{aligned} w^*(x) &= \int_0^x f_x^+(\tilde{x}, \tilde{x}) d\tilde{x} + w_0 \\ &= \int_0^x \left(\alpha\theta\tilde{x}^{2\theta-1} + h_x(\tilde{x})\right) d\tilde{x} + w_0 = \frac{\alpha}{2}x^{2\theta} + h(x) - h(0) + w_0. \end{aligned}$$

Under f^-, the equilibrium wages are

$$\begin{aligned} w^*(x) &= \int_0^x f_x^-(\tilde{x}, 1-\tilde{x}) d\tilde{x} + w_0 \\ &= \int_0^x \left(\alpha\theta\tilde{x}^{2\theta-1} + h_x(\tilde{x})\right) d\tilde{x} + w_0 = \frac{\alpha}{2}x^{2\theta} + h(x) - h(0) + w_0. \end{aligned}$$

Under both functions the wages on the equilibrium path are exactly identical, and from wage data alone one cannot distinguish between positive and negative sorting. The problem is obtaining the order of the firms. If we only have wage data and no profit data, and we derive the order on the firms by ranking them by increasing wages, we will obtain two different orders depending on whether we have complements or substitutes. To see this, observe that under positive assortative matching (henceforth PAM) higher type firms pay higher wages along the equilibrium path whereas under negative assortative matching (NAM) higher type firms pay lower wages. In the former $w(y, y) = \frac{\alpha y^{2\theta}}{2}$ is increasing in y, in the latter $w(1 - y, y) = \frac{\alpha(1-y)^{2\theta}}{2}$ is decreasing in y. This result is true for any general production technology as summarized in the proposition that follows below.

Figure 1 has an example with the profits, wages and total output when $f^+ = xy + y$ and $f^- = x(1 - y) + y$. Observe that wages are identical in both cases (blue), but that profits are decreasing in worker type x under f^+. While higher y firms have higher profits, higher x workers are matched with lower y firms who obtain lower profits. In the example, with $w_0 = 0$, we obtain $w^+(x, \mu(x)) = \frac{x^2}{2}, \pi^+(x, \mu(x)) = \frac{x^2}{2} + x, f^+(x, \mu(x)) = x + x^2$ and $w^-(x, \mu(x)) = \frac{x^2}{2}, \pi^-(x, \mu(x)) = \frac{x^2}{2} + 1 - x, f^+(x, \mu(x)) = 1 - x + x^2$.